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Sensitivity Analysis: A Technique for Investigating the Impact of Changes in Project Variables

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Abstract
The discounted cash flow technique is referred to as time –adjusted technique in that the analysis takes into consideration the effect of time on the value of money. This is the uncertainty of future returns and the effect of inflation. The approach makes use of a discount factor, which reduces the cash inflows and cash outflows to their present values. These cash inflows and discount factors are subject to change because most future development projects carry with them some speculative and subjective input variable. The quantity and quality of some factors of cash inflows and outflows that are implicit or even circumstantial to the execution of the project are often tuned left in the realm of guesswork. Sensitivity analysis is a major approach to re-examining an already concluded viability study in order to determine what the investment appraisal outcome would be, if same or all the factor elements were to vary. This study therefore, discussed the Discounted Cash-Flow Techniques – the NPV & IRR, a practical example of sensitivity analysis of effects of changes in key variables. The recommendations include – entering into an agreement of long term supply contracts at specified quality and prices to reduce uncertainty of operating costs.

Introduction

Cash flow analysis according to Iroegbu (2004) and Nwanekezie (2009) consists of cash inflows or cash outflows as they affect a particular business or investment. Cash inflows are the capital or revenue cash coming into the business or project. Cash outflows refer to capital expenditure or periodic outgoings involving the movement of cash out of the business or project. The concept of cash inflows and cash outflows can equally be expressed by the notion of positive cash flows and negative cash flows (Umeh, 1977). Basically, there are two approaches of cash flow analysis. These include crude cash flow or time unconscious cash flow analysis and discounted cash flow technique, which take into account the effect of time on cash flow.

Discounted Cash-Flow Technique

Johnson (2000) opined that discounted cash flow technique involves the discounting of all future receipts and expenditures similar to the investment method of valuation, but they can readily be used to allow for inflation, taxation and frequent charges in the amount of income and receipt as may be required. Ifediora (1993) defined it as cash flow which had been discounted at a given rate percentage in order to take account of the passing of time. Umeh (1977) is of the view that the discounting cash technique is essentially a valuation technique all though the presentation may differ from that of orthodox valuation.

Discounted cash flow is necessarily a technique that reduces the various cash flows to their present value equivalent using a discount factor. The discount factor or rate is commonly arrived at by adopting a risk-free rate and making allowances for the risks associated with the particular property. The discounted cash flow technique introduces the concept of present value in order to reduce to manageable proportion, the time – dimensions involved in the investment project analysis. It is expressed by:

$$Pv = \frac{1}{(1+i)^n}$$

Where:

Pv = the present value of N1.
i = the discount factor.
n = the time element in months or year

The valuation involves the use of present value of N1 per annum or year purchase. On the other hand, if the cash flows are of divergent magnitudes from time to time, individual cash flows are discounted using the present value of N1 (Ifediora, 1993).

The two most commonly used methods according to Iroegbu (2004), Nwanekezie (2009) and Umeh (1977) include:

i. The Net Present Value (NPV).

ii. The Internal Rate of Return (IRR).

The Net Present Value (NPV)

This method is used to determine the sum of money representing the difference between the present value of all inflows and all outflows of cash associated with the project, by discounting each at a target rate (Udo, 2003). The technique uses an external rate of return or discount factor, which is the rate an investor is expecting from a particular investment or an assumed discount factor based on the market survey and analysis of the performance of similar investments in the same investment environment.

Viewed as the opportunity cost of money, it is based on the equivalent yield of long-dated government stock with a percentage addition or a discount to reflect the nature of the project. It is the minimum rate of return which the investor requires in order to make the investment worthwhile, taking into account the risk involved and all other relevant factors.

The Net Present Value Method has advantage over the one-stage YP model, which is its ability to calculate the present value of cash flows that do not conform to either constant annuity, arithmetic, or geometric growth or decline. The presence of net positive present value indicates not only that the cash inflows are large enough to cover the original capital tied up in the investment and also that the project would earn the desired rate of return but would also earn bonus revenue over all above the required rate of return (Umeh, 1977).

Profitability of project tends to be affected more by choice of target return than the quality of investment. However, the greatest problem with the net present value method of discounted cash flows is the choice of appropriate external rate of return applicable to a particular project or investment.

For examination of the discounted cash flows technique using net present value method, the cash flow projection for the expansion of Tarbas Memorial Comprehensive High School would be used. Tarbas Memorial Comprehensive High School was established in 1985 and has come to stay as one of the foremost private post primary institutions in Ikono Local Government Area of Akwa Ibom State. This institution is a household name in the area and is conspicuously located at Ibiaku Ntole Okpo, the Local Government Head-quarters of Ikono council.

The 6.628 hectares ground of this institution houses 7 units of staff quarters, 4 extensive classroom blocks, technical workshop, assembly hall, administrative building and girls' hostel. Under construction are boys' hostel and dinning hall. Sporting arena include football field, volleyball pitch and table tennis court. The undeveloped portion of the school complex is planted with palm trees and assorted economic trees.

The authorities and management of the institution were considering an expansion of the college facilities and structures, rehabilitating the existing structures, designing and construction of ancillary structure, and provision of infrastructure thus, commissioning a firm of Estate Surveyors & valuers to examine the feasibility and viability of the prospects. The appraisals used the cash flows analysis to examine the project. Assessment was done of the revenue (income) accruable to the investment and the cost of maintenance of the project so as to continue earning income. The studies indicated a targeted return of 15% and the cash flow forecast would be used in analyzing the effect of changing cash flows and discounted factors on viability studies. The various cash inflows (revenue accruing to the institution as the result of the expansion) and cash outflows (cost of expansion and its operating and maintenance cost) are as indicated in table 1 below.

Table 1
Discounted Cash Flow Technique-Net Present Value Method (Base Case)

YEAR	CASH INFLOW	CASH OUTFLOW	DISCOUNT RATE @ 15%	DISCOUNTED CASH INFLOW	DISCOUNTED CASH OUTFLOW
0	-	73,239,000	1	-	73,239,000.00
1	16,600,000	20,125,000	0.8696	14,435,360.00	17,239,000.00
2	16,600,000	5,355,000	0.7561	12,551,260.00	4,048,915.00
3	28,720,000	6,426,000	0.6575	18,883,400.00	4,225,095.00
4	28,722,000	6,426,000	0.5718	16,422,096.00	3,674,386.80
5	28,720,000	6,426,000	0.4972	14,279,584.00	3,195,007.20
6	36,600,000	7,711,200	0.4323	13,228,380.00	3,333,551.76
7	30,600,000	7,711,200	0.3759	11,502,540.00	2,898,640.08
8	30,600,000	7,711,200	0.3269	10,252,995.20	2,520,791.28
9	36,064,000	9,253,400	0.2843	8,915,020.80	2,630,741.62
10	36,064,000	9,253,400	0.2472	7,750,152.60	2,287,440.48
11	36,064,000	9,253,400	0.2149	7,750,153.60	1,988,555.66
	-	-	-	138,223,929.60	121,542,825.40

Total discount cash inflow = N138, 273,929.60
Total discount cash outflow = N121, 542,825.40
Net positive present value (NPPV) = 16,681,104.20

At a discount rate of 15%, the project shows a net positive present value of N16, 681, 104.20 after eleven years, which makes it profitable on the face of it for that period and means that the investor will realize a percentage return over the target rate within this time frame.

The use of high rate of return would reduce the viability of the project and the use of low rate increases the viability. The effect of changing discount factor or rate, as well as changing cash flows shall be tested using the sensitivity analysis.

Internal Rate of Return (IRR)

This is the discount rate that equates the discounted cash inflow to the discounted cash outflow (Ogbuefi, 2002). The internal rate of return avoids the

shortcoming of the net present value method by using only the rate of interest as the means of measurement. It is the investment yield. It is found by trial and error by applying present values at different rates of interest in turn to the net cash flows. In ranking of projects, the investment Criterion is that the Internal Rate of Return (IRR) should be at least as much as, if not more than the opportunity cost of the capital invested, assuming similar degrees of risk.

Internal Rate of Return according to Iroegbu (2004) and Nwanekezie (2009) may also be seen as the higher rate of interest at which an investment can be funded if the cash flow generated is to be sufficient to repay the original investment at the end of the project life, having regard to any interim payments on capital required, where appropriate, at any time together with an annual surplus to reflect the degree of risk in the investment.

For the analysis of Internal Rate of Return (IRR) method, the same cash flows in table 1, would be used. At 15% discount rate, the following data were gathered:
Present Value of Cash Inflows was N138, 223, 929.60
Present Value of Cash Outflows was N121,542,825.40
Net Positive Present Value of N16,681,104.20.

However, the NPV in this case is relatively high, meaning that the IRR should be greater than 15%. Using trial and error, it becomes necessary to use an IRR that would be higher than 15%. The essence is to find an IRR where NPV would be negative. When this is achieved, the two IRR would be interpolated using the formula:

$$IRR = R + \left\{ R_2 - R_1 \frac{NPVR_1}{NPVR_1 + NPVR_2} \right\}$$

Where R_1 represents the smaller of the two discount rates.

Another trial of a higher rate at 18.75% using the same cash flows would give the following figures:
Present Value Cash Inflow =N116,941,143.20
Present Value of Outflow = N115,817,168.60
Net Positive Present Value =N1,123,974.60.

Here again, the cash inflow is greater than the cash outflow resulting in net positive present value, but this time of lower figure. This means that the IRR is much closer to the discount rate of 18.75%.

Another trial at 19.13% would give the following data:
Present Value Cash Inflow =N115,066,345.60
Present Value of Outflow = N115,308,860.60
Net Negative Present Value =N242,515.00
The negative NPV indicates that the IRR is below the discount rate of 19.13%.

To find the actual IRR, the earlier stated model will be applied to the discount rate of:
R_1 = 18.75%

R_2 = 19.13%

Thus:

IRR =0.1875 + (0.1913 – 0.1875) $\frac{1,123,974.60}{1,123,974.6+242,515.00}$

IRR = 0.187 + (0.0038) (0.8225)

IRR = 0.1875 + 0.0031

IRR = 0.1906 = 19.06%

Therefore, the internal rate of return for cash flows in table 1 is 19.06%. if, there is variation or change in cash inflow or cash outflow, would the IRR or NPV be distorted. This and others would be examined using sensitivity analysis.

Sensitivity Analysis

This is the process by which changes in variables (output) caused by changes in parameters can be ascertained. Here parameters values do not concide with requirements of the design. A major result of sensitivity analysis is on increased awareness of factors, which have major effects on the design. According to Adindu (2009) sensitivity analysis may result into reformation of the system model.

Sensitivity analysis shows to what extent the viability of a project is influenced by variations in major quantifiable variables, while the risk analysis considers the probability that changes in major quantifiable variables will actually occur. According to Umeh (1977), a sensitivity test is essentially a reappraisal of a project before its commencement using different behavioral criteria or value dimension for significant variable elements. Parameterized in the first instance, in order to arrive at an optimal solution or certain conclusions as to the viability of a project or scheme appraised. It tests how robust the conclusion of a viability appraisal is and shades important light on which variables exert greater individual (or combined) influence on the conclusion of a variability appraisal and also, to what extent?

The financial and economic benefit – cost analysis of educational project is based on forecasts of quantifiable variable such as demand, costs and benefits. The values of these variables are estimated based on the most probable forecasts, which cover a long period of time. The values of these variables for the most probable outcome scenario, are influenced by a great number of factors and the actual values may differ considerably from the forecasted values, depending on future developments.

It is therefore, essential to examine the effects of likely changes in the major variables on the viability of the project. The variability of the project is evaluated based on a comparison of its Internal Rate of Return (IRR) to the financial or economic opportunity cost of capital. Alternatively, the project is considered to be viable when the Net Present Value (NPV) is positive, using a selected discount rate. Sensitivity analysis, therefore, focuses on analyzing the effects of changes in key variables on the projects IRR or NPV, the two most widely used measure of project worth.

In economic analysis of the college, there are also other aspects of project analysis. These according to Nwanekezie (2009) include:

1. **Demand Analysis:-** This is to assess the sensitivity of the demand forecast to changes in population growth.

2. **Least Cost Analysis:-** This is to verify whether the selected least – cost alternative remains the preferred option under adverse conditions.

3. **Sustainability Analysis:-** This is to assess possible threats to the sustainability of the project.

Sensitivity analysis is particularly concerned with factors or combination of factors that lead to unfavorable consequences. These factors would normally have been identified in the project (Logical) framework as "project risks or "project assumptions". Sensitivity analysis tries to estimate the effects on achieving project objectives if certain assumptions do not, or only partly, materialize. Risk analysis assesses the actual risk that certain assumptions do not or only partly, occur.

The Purpose of Sensitivity Analysis

Sensitivity analysis is a technique for investigating the impact of changes in project variables on the base case. The purpose of sensitivity analysis as listed by the studies conducted by the Asian Development Bank (ADB) in 1999 for Economic and Resources Centre include:

i. To help identify the key variables which influence the project cost and benefits streams. In the case of the college, key variables to be normally included in sensitivity analysis are educational demand (number of students seeking admission into the college), investment cost, operation and maintenance cost of the project, financial and economic revenues, and discount rates.

ii. To instigate the consequences of likely adverse changes in these key variables.

iii. To asses whether project decision are likely to be affected by such changes.

iv. To identify actions that could mitigate possible adverse effects on the project.

Execution of Sensitivity Analysis

To meet the above purpose, the following steps according to Nwanekezie (2009) would be followed:

i. Identify key variables to which the project decision may be sensitive.

ii. Calculate the effect of likely changes in these variables on the base – case IRR or NPV, and calculate a sensitivity indicator or switching value.

iii. Consider possible combinations of variables that may change simultaneously in an adverse condition.
iv. Analyze the direction and scale of likely changes for the key variables identified, involving identification of the sources of change.

Practical Example

Step 1: Identification of Key Variables:- The base case project economic analysis incorporates many variables, quantities and their inter-relationships, prices or economic values and the timing of project effects. Some of these variables will be predicable or relatively small in value in the project context. The following, according to Nwanekezie (2009) are the likely key variables that would influence the investment decisions:

i. Variables which are numerically large such as investment cost, and projected cash inflow (revenue from school fees).
ii. Essential variables, which may be small, but the value of which is very important for the design of the project. For instance, assumed income and population growth.
iii. Variables occurring early in the project life, such as investment costs, which will be relatively unaffected by discounting.
iv. Variables affected by economic changes, such as changes in real income.

Step II: Calculation of Effects of Changing Variables

The values of the base indicators of project viability (IRR and NPV) should be calculated for different values of key variables. This could be done by calculating sensitivity indicator and switching values.

A sensitivity indicator towards the Net Present Value (NPV) compares percentage (%) changes in IRR above the cut – off rate with percentage change in a variable or combination of variables. Switching value towards percentage change in a variable or combination of variables to reduce the NPV to zero. Switching value towards the Internal Rate of Return (IRR) is the percentage change in a variable or combination of variables to reduce the IRR to the cut – off rate i.e. discount rate. The sensitivity indicator towards NPV is expressed as:

$$SI = \frac{(NPV_b\ NPV_1)/\ NPV_b}{(x_b - x_1)/\ x_b}$$

Where:

x_b = Value of variables in the base case.
X_1 = Values of the variable in the sensitivity test
NPV_b = Value of NPV in the base case.

NPV_1 = Value of the variable in the sensitivity test.

The sensitivity indicator (SI) towards the IRR is given by:

$$SI = \frac{IRR_b - IRR_1 / (IRR_b - d)}{(x_b - x_1)/ x_b}$$

Where:

x_b = Value of variable in the base case.
x_1 = Value of the variable in the sensitivity test.
IRR_b = Value of the variable I the sensitivity test.
d = Discount rate.

The switching value towards the NPV is given by:

$$SV = \frac{(100 \times NPV_b)}{(NPV_b - NPV_1)} \quad \frac{(X_b - X_1)}{X_b}$$

Where:

X_0 = Value of variable in the base case.
X_1 = Value of the variable in the sensitivity test.
IRR_b = Value of IRR in the base case.
IRR_1 = Value of the variable in the sensitivity test.
d = Discount rate.

Calculation

Sensitivity indicator towards NPV-Base case:

Cash Inflow = C_b = N319, 352, 000.00; NPV_b = 16,681, 104.20

Scenario 1:

C_1 = N287,416,800 (10% change)
NPV_1 = N2,858, 711.20

(16,681, 104.20 – 2,858,711.20)/16,681,104.20 (319,352000 – 287,416,800)/319,352,000

$$SI = \frac{0.8286}{0.1} = 8.29$$

Sensitivity indicator towards IRR – Base case:

Cash inflow = C_0 = N319,352, 000

IRR_b = 19.06%

Scenario 1:

C = N287, 416,800 (10% change)
d = 15%

$$SI = \frac{(O.1906 - 0.1581)/ (0.1906 - 0.15)}{(319,352,000\text{-}287,416,800)/319,350,00}$$

SI = 8.00

Switching value towards IRR – Base case:

Cash Inflow = C_1 = N319,352,000

IRR_b = 19.06%

Scenario 1:

C1 = N287,416,800 (10% change)

IRR, = 151 81%

d = 15%

$$SV = \frac{(100x10.1906-0.15)}{(0.1906 - 0.1581)} \quad X \quad \frac{(319,352,000-287,416,800)}{(319,352,000)}$$

SV = 12.49%

Interpretation - Sensitivity Indicator

Percentage change in NPV respectively and percentage change in IRR above the cut –off rate (15%) is larger than percentage change in the variable. This means that cash inflow is a key variable for the project.

Switching Value

A change of approximately 15% - 20% in the cash inflow variable is necessary before the NPV becomes zero or before the IRR equals the cut-off rate.

Characteristic

The sensitivity indicator shows which variables the project is or is not sensitive to and suggests further examination of change in variable. The switching value measures the extent of change for a variable which will leave the project decision unchanged. The switching value is, by definition, the reciprocal of the sensitivity indicator.

Sensitivity indicators and switching values calculated towards the IRR yield slightly different result if compared to Sts and SVs calculated towards the NPV. This is because, the IRR approach discounts all future net benefit at the IRR value and the NPV approach at the discount rate d. In the base case, the NPV is N16,681,104.20 and IRR is 19.06%. The sensitivity of the base case according to Umeh (1977), Iroegbu

(2004) and Nwanekezie (2009) has been analyzed for adverse changes in several key variables as follows:

i. An increase in investment cost by 20%.
ii. A decrease in cash inflow by 20 %.
iii. An increase in costs of operation and maintenance by 20 %.

The effects of the above changes are summarized in cash table 2 below:

Table 2
Sensitivity Analysis: A Numerical Example

ITEM	CHANGE	NPV	IRR%	ST(NPV)	SV(NPV)
Base case		16,681,104.20	19.06	-	-
Investment cost	+20%	-1,466,835.80	11.90	5.44	18.38
Cash inflow	-20%	-10,963,681.70	13.23	8.29	12.07
O & M cost	+20%	10,520,443.30	18.09	1.85	54.15

Where SI = Sensitivity indicator
SV = Switching value
O & M = Operation and maintenance cost

Source: Based on data in table 1.

Analysis of Effects of Changes in Key Variables

In the case of an increase in investment cost (cost of expansion) of 20%, the sensitivity indicator is 5.44. This means that the changes of 20% in the variable (investment cost) results in a change of (5.44x20%) = 108.8% in the NPV. It follows that the higher the sensitivity indicator the more sensitivity the NPV is to the change in the concerned variable.

In the same case, the switching value is 18.38% which is the reciprocal value of the SI x 100. This means that a change (increase) of 18.38% in the key variable (investment cost) will cause the NPV to become zero. The lower the SV, the more sensitivity the NPV is to the change in the variable concerned and the higher the risk with the project.

In the example of decrease in cash inflow of 20%, the sensitivity indicator is 8.29 and the switching value is 12.07. The sensitivity indicator of 8.29 means that there is 165.8% changes in the NPV and thus affects the variability of the project. Also, the SV of 12.07% means that the project is sensitive to variation (decrease) in cash inflow.

Also, in the case of an increase in operation and maintenance cost of 20%, the sensitivity indicator is 1.85 and switching value is 54.15%. It can be inferred that a change of 20% in operation and maintenance cost would result in 37% change in NPV.

Thus, the operation and maintenance cost is less sensitive to the project, if there is increase in the variable by 20%. It follows also that the variable is less sensitive to the project. At this point, the results of the sensitivity analysis should be reviewed. It should be asked:

i. Which are the variables with high sensitivity indicator?
ii. How likely are the (adverse) changes (as indicated by the switching value) in the values of the variable that would alter the project decision?

Conclusion

Investment project takes time to be implemented and the longer it takes to be executed, the more the need to rely on forecast figure and consequently, the more the need for sensitivity test or analysis. The changing nature of cash flows and discount factor underscored the imperative of sensitivity analysis in any project. Sensitivity analysis is useful at all stages of project processing at the design stage to incorporate appropriate changes; at the appraisal stage to establish a basis for monitoring, and during project implementation to take corrective measures. The uncertainty surrounding the results of the economic and financial analysis is expected to decrease as the project moves in to the operational phase.

Recommendations

Steps should be taken to reduce the extent of uncertainty surrounding the variables where possible. The remedial actions suggested include:

1. Specific agreements should be made to ensure contractor performance and project quality during construction works to reduce the likelihood of delays.
2. An agreement of long-term supply contracts must be entered at specified quality and prices to reduce uncertainty of operating costs.
3. Capacity building activities must be formulated to ensure appropriate technical and financial management for the running of the school.
4. Information, awareness or education programmes must be conducted to ensure the involvement and awareness of the dwellers of the neighborhood of the school curriculum.
5. School fees and other sources of income must be adjusted to ensure sufficient revenue.

(6) Technical assistance programes must be conducted to develop appropriate incentives to encourage higher level of productivity.

(7) A pilot scheme must be implemented to test technical assumptions and observe user's reactions.

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