PIANOFORTE STRINGS AND THEIR PROPER DIMENSIONS

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Abstract

The necessity of acquiring some knowledge of the principles of mechanics* before proceeding to the study of scale design, admitted. Such knowledge together with that of the principles of acoustics as they apply to musical sounds produced by vibrating string is essential for a correct understanding of the fundamental ideas underlying true pianoforte design. To know a piano accurately, one must understand the laws governing the tone quality, and how the propagation and transmission of sound is produced as well as the pitch and intensity of sound. As there are thousands of scholars today who are anxious to obtain information, and more correct knowledge, the author of this article therefore dwelt on this topic and believes that it may be read and comprehended, even by one to whom the very term acoustics has hitherto been unfamiliar.

The strings of a modern pianoforte are made of cast steel and posed a relatively great thickness and stiffness. That is to say, they enjoy these characteristic to a far greater degree than do the string of any other musical instruments that employ such agents for the purpose of generating musical sounds. The string of any member of the viol family, for example, are so totally unlike those of the pianoforte that no compression of their respective behaviour when subjected to tension can be of interest to any save the scientists.

In dealing with the strings of the pianoforte then, the author faces an isolated and unusual problem, which we shall have to consider at some length. We shall investigate the peculiar effects produced by the high tension, great thickness and great stiffness of the string, as well as the singular phenomenon exhibited in the case of the covered bass string. We shall note that the strings are responsible for many unpleasant things which they are seldom accused, of and their proportions as top length and tension do not comprehend in themselves the whole problem that the sealing presents to the designer.

As is generally known, the strings that are charged with the duty of emitting the sounds comprehended within the two lowest octaves on the pianoforte are customarily constructed of a combination of steel wire and some others, usually copper iron. The copper is wound over a core of the former wire and these winding graduates, as the amount and thickness of the material employed according to the pitch to which it is desired that each string shall be tuned. There is an obvious reason for this procedure. For example, two strings whose lengths are 2:1 will, other things being equal, emit musical sound separated by the imperial of an octave. Consequently, under perfect mechanical conditions, the length of each strings of a pianoforte should conform to the rule thus, indicated and should be on one-half or double the length of that which produces the octave about or below it, the absolute application of this rule, however, being subject to certain practical modification throughout the entire compass. These will be discussed later.

Even in the absence of such consideration, however, this deal condition could not be attained. The mechanical difficulties present would always be operated to forbid the carry out of such an arrangement throughout the whole compass of the instrument. For, to follow the rule with entire consistency would necessitate a length of 256 inches for the lowest C; on an assumed length of 2 inches for the highest note of the instrument of nearly 24 feet it is not difficult to see that such construction is impossible. Furthermore, evenness of tone equality would be seriously hindered if the lowest strings were of any such dimension. To secure equality of tonal result it is necessary, as has been noted above, to equalize as far as possible, the particular forms of vibration that pertain to each strings. Obviously, the nature of the below that would produce a given form of vibration in a string of 256 inches in length must be very different from that which would produce similar forms in a string only one tenth (1/10), long. Again, to maintain such a long string at the required tension involves mechanical problems that save more of engineering than of pianoforte building.

For these and cognate reasons, therefore, the practice has arisen of artificially slowing the rate slowing the rate of vibration in the bass strings by wrapping them with brass, iron and copper wire.

Naturally, the forms of the vibrations cited in these wrapped strings are entirely different from any that the plain steel wire is capable of producing. The iron or copper wire is itself thrown into vibration both independently of and together with the cord of steel, so that we have the phenomenon of one string emitting two separate series of vibrations, with resultant disarrangement of the generated upper partials and concomitant production of beats in a more or less appreciable quantity. Now if in addition, the bass strings are not sealed with approximate correctness as to their relative lengths, thickness, and other dimension, it follows that there will be two distinct and different causes-, of dissonance and unevenness of tone-quality, either of which is sufficient, in itself, to produce very unpleasant tonal result. It is clear, then that particular attentions must be paid to the designing of the string arrangement, if excellence, if excellence of tone-quality is to be anywhere approached.

It is, fortunately, possible to give quite precise direction for the calculating of string dimension. As a preliminary, the author must remind the reader of the rule that is relating to the behaviour of stretched strings, in "Acoustical laws of a sounding string". It will be recalled that the author had occasion to observe that these rules would require certain modifications in practice as they referred only to ideal musical strings which are at perfect flexibility and perfect uniformity and are stretched at an absolutely constant tension.

The first modification that appears upon investigation has reference to the division of string-lengths. It has already been pointed out that, in practice, the octave the fundamental tone at a given pianoforte strings by dividing it exactly in the middle. Conversely, an exact doubling of the length does not produce below the given fundamental tone.

This discrepancy occurs on account of the fact that the shortening or lengthening of a given string causes a corresponding change in the tension at which it is maintained and in the density of adhesion of its molecules.

Now the length of a string is doubled, in order to obtain the octave below its fundamental tone, we decrease resilience of the string brought about by the lends also to decrease the frequency. The frequencies of vibration of string vary directly as the thickness, and directly also as the stiffness. These axioms being admitted, it is observed that to obtain an octave lower than a given fundamental must be obtained. Therefore, as it is seen from above, one-fourth to all for the automatic decrease of stiffness must decrease the double length, which varies directly as the frequency. And this modification must itself be modified to compensate for the increase in frequency produced by the vary act of shortening. Therefore, the tension must be considered and be finding that to reduce this tends again to decrease the stiffness in exactly the same proportions as it was before increased. But frequency of vibration varies as the square root of the tension; therefore, we take the square root of one-fourth which was the fraction first arrived at. This root is one-sixteenth and is the differential factor must be subtracted from the ideal octave length in order to obtain the practical length.

It will be found that the differential factor here suggested does not provide a complete solution to the problem of allowing for the exhibited differences between theory and practice. It does, however, provide a true guide to the lengths. There is of course a difference of produced frequency to be allowed. Fortunately, however, this is provided for by the graduate thickness of pianoforte wire. By taking advantage of this almost geometrically proportional diameter we are able to calculate a stringing scale that, if adhered to, will give the nearest possible approximation to complete harmony between theory and practice. That is to say, we can proceed with a string length calculation based upon the differential factor already obtained, and then by arranging the distribution of the string thickness to the diameters that are provided by the manufacturers of music wires, we may obtain a true estimate not only as to the thickness of wire to be used at each place, but also as to the length proper to each string. Of course the reader of this will remember that the matter of pitch is of considerable importance in all calculations of this kind. A difference in pitch implies difference of tension when the other factor remains equal, and we therefore have calculated the following tables on the assumption that the pitch to be used is that known as the international or C517. Attention is there, directed to the following:

Table swing true length of octave strings from the highest "c" string to the last "c" string that is usually left unwrapped.

C5 =2.048 IN2-/23: APPROX.

C4	=2.048 x 1.9375= 3.968 in- 3 24/23
C3	=3.968 x 1.9375-7.688 in = 7 17/23:
C2	=7.688 x 1.9375 = 14.875 in ~ 14 7/8:
Cl	=14.875x 1.9375 = 28.820 in - 28 4/5:
	=28.820x 1.9375 = 55.828 in = 55 4/5:

NOTE - The length of the first string is chosen arbitunity, but as given is a very close approximation to the practice of the best American makers. Vulgar fractions are calculated from the decimal and the error in no case exceeding about one-fiftieth of an inch. The differential factor is, as we know, 1/16 therefore 1 15/16, or we multiply by (2-1/16) in decimal 1.9375).

The above table, then, affords us a reliable guide to the scaling of the unwrapped strings. At the same time, however, it is not by any means complete for the reason that there is no method shown as yet for the calculation of the other and intermediate string-lengths. The author is however, able to accomplish this task by the aid of a very ingenious rule proposed by the later professor pole, F.R.S. As follows:

The proper length of any string provided the length and frequently of this second string be known. Given these factors: then:

1. Take the logarithm of the known string
2. Multiply the number 025086 by the number of semitones that the sound to be given by the required string length is above or below the sound produced by the given string.
3. If the required string is below the given string, add together the two numbers obtained, if it is above the string, subtract the second number from the first, the result in both cases in the logarithm of the required length.

For example, we have calculated already the proper length of C. in hundredths of an inch. This length is expressed as 2882. The logs, of this number are 45943. (This may be verified by the use of any table of logarithms).

It is required to obtain the length of the string that, caeter Paris will produce one semitone above C. 45945 - LOG. Of 2882 02508 (6) = .2508 (6) x the number (1) at semitones that required string sound above given string.

By substation, 43435 = log. of 2718 = length of required string in hundredths of an inch; Required length for C - sharp =27 18/100 inches b y reversing the process describing above, and adding instead of subtracting the proper lengths for the semitones below and all other in descending progression may be calculated with accuracy.

Having thus settled the matter of string length, we may proceed to consider the question of diameter. But it is first of all necessary to warn the reader of this that the length that have been calculated refers only to such pianofortes as are capable by reason of their size of taking the idea string length of very small upright pianos. An example, this cannot be brought within that class except as regards the highest of their string.

In all pianofortes, no matter what the size, the higher strings are practically identical in length, but it will be found that shortness of highest in an upright or of length in a grand begins, towards the middle of the scale disastrously to affect the string proportions. As already pointed out, there are only two ways these proportions can be overcome. These are through alterations in the rotation to the thickness, but such alterations necessarily disturb the total balance, and here we find a very strong reason for the poor tone that the average atrophied grand or upright possess as moreover, it must not be forgotten that disproportionate thickness or unduly slackened tension affects the actual nature of the vibrations that are set up within the string. And the affections are operative both as to frequency and to form; hence they are naturally bad tones, and inability to stand in tune.

This is not intended as an argument against the small pianoforte, but it is desired here to show that these little instruments, whether horizontal or vertical, must not be expected to perform impossibilities. If we are obliged to build small instruments, we must revise our calculations and tabulate the string-length according to the different basis at apportionment. For the purpose of the present work, however, the calculations have made on the assumption that the standard size of pianoforte is designed. Then in consideration of the strings diameter, the cast steel wire that is used for the pianoforte string is supplied in definitely numbered and graded thickness. The numbers that are used generally range from No. 13 to No.24. According to the tests made at the Chicago worlds fair by the aid of riehle Bros, testing machine, the wire of these numbers were of the following diameters and broke at the following strains. The wire manufactured by the firm of Mortise Poehlman, Nuremberg, Germany, has been selected from among the various products that were subjected to these tests, on accounts of its superior durability and evenness of graduation

Numbers	Diameter in fraction of an inch	Broke at of ibs
13	.030	325
14	.031	335
15	.032	350
16	.035	400
17	.037	415
18	.040	
19	.042	
20	.044	

Now is a well known fact, and indeed obvious from what has already been said that the proportion relation as to length, tension, diameter and breaking strain does not permit any other arrangement for the sealing of wire that which is universally accepted. The is to say, the shortest wires are taken from the thinnest numbers, and vice-versa, the whole scaling being so arranged as to secure for each tone which string shall be stretched at approximately the same tension. Experience and the observations of the most eminent manufacturers seem to have established that the strain upon each of the uncovered strings should be maintained, as nearly as possible, at 160 ibs. If this be done it will be found that a pianoforte so constructed in the table referred to. It wills of course be necessary to arrange with due proportion the number of strings that are to be taken from the wire of each number. It will be found that the best practice takes into account the half sizes not shown here and strings the instrument with an average of five tunes to each thickness of wire, beginning at 13 or 131/2 and counting down to the end of the unwrapped strings according to the general directions digested. Experience and the individual ideals of the designer by such knowledge as this work aims to impact, are the best guide that can be followed. Empirical induction, based upon observation and experience, provides the only possible and practical means for arriving at the true and proper arrangement to be made for each individual instrument. This empiricism extends with particular force to all string arrangements and is seen nowhere so conspicuously as in the barite of methods that are adopted by piano manufacturers in determining the numbers of strings within the unwrapped section of the scale. Thus certain makers carry the wrapping over to the beginning of the table strings and have two or three string-groups provided with wrapped wire before the over-stringing is begun. The idea here is either to correct original defects of scale design or to shade down the break in tone that so often occurs at the point where the over-string usually begins. From observation of the best makers, it may be said that the tone C below middle C is usually the first over string tone. Of course when the instrument is very small it will often be found that it is impossible to give the unwrapped strings the proper lengths. In his case these offending strings may either be covered with light wrapping or may be put bodily over into the string portion of the scale, in which later will be wrapped.

Supposing then that the matter of the number of overstrains strings has been determine, we may proceed to the consideration of the dimensions, number and covering of the strings that are to serve here. The author is obliged here to confess that the problem of attaining good tone m the bass is, indeed difficult. It is by no means hopeless, however, as the success of the more than one eminent maker as already demonstrated

The simplest, most obvious, and easiest way out of the inherent difficulties of the scaling of bass strings is to be found in the consideration of their proper lengths. It does not require very much thought to perceive the truth that the longer the strings, the less the weight needs be too imposed upon them. If we make the bass strings to approach as far as may be the lengths that they would required to have if unwrapped, we shall be able to reduce proportionately the amount of artificial control that has to be exercised over the vibration speed. Not only this, but the greater length thus attained implies greater tension. That is to say that as, we saw before the tension at which a string is stretched acts to overcome the slowness at vibration speed induced by its greater lengths and consequently, tends to generate a more regularly progression of the upper partials (as experiment has demonstrated), with resultant tendency to greater purity of tone quality.

It may be accepted as an axiom that the bass strings should be as, simultaneously, as lightly weighted as possible, and that the weight of them should be strictly proportional to the pitch of the musical tone that they are desired, at a given tension to emit. As far as the second clause of these conditions is concerned, it is well to remind the reader of this article that limitations of space within the body of a piano usually determine the possible length of the bass strings. So much so is the case, that it is not often possible to make any greater difference in their respective lengths. The best makers of piano appear to have agreed in a method of treating this problem that is at once simple and effective. They recognize the great advantage of scaling the bass string at the greater possible length and then, they make the lowest bass string one-fifth longer that the highest. At the same time, the so graduated weight of the wrapping material that the same result are attained as would naturally follow if they were as accurately scaled, in proportionate length as are the plain wire strings.

The equalization only approximates, for the forms of vibration excited in two strings of the same pitch, will be different whenever the variable. Thus when the factor of length is varied no counter-adjustment of tension, thickness or density can restore to the strings so modified, the exact form of vibration that may have originally possessed, consequently it becomes impossible to induce from artificially weighted strings precisely the same series of partial tones that a plain wire filament will emit, even when the tones generated by the two strings are of the same pitch.

Conclusion

The lesson of this is plain. As perfection of tonal quality can only be attained impact, it especially makes to pay strict attention to each scaling of the bass string which will furnish complement of sound producing agencies that may be relied upon to induce as nearly as possible the same successions of partials as are habitually emitted throughout the higher sections or the piano. Thus, it becomes evident that greater practicable length and the least practicable weight are the chief factors that must govern the designer in laying out the scale for the bass string. The relative densities of the wrapping materials some played in the manufacture of bass strings have been the subjects of consideration study. Brass, which was the earliest subject of experiment, has long been superset by either copper of iron. As to the relative advantages posed by the two materials used in wrapping strings, it .can be said that once the chief and almost the only advantages presented by the later lies in the relative cheapness. Acoustically, however, copper from the most suitable materials for the winding of bass strings and this force the following reasons.

The specific gravity of copper is 8.78, while that of iron is 7.78. Again, the former metal, while inferior in tenacity to the later, possesses the great advantage of higher ductility, so that its classic qualities are marked. It is thus evident that copper is a more suitable material for the generation of music sound than iron. And the qualities, which we have just noted as pertaining to it, are precisely those most useful in the production of harmonic processions of partial tones. It is therefore clear that as between copper and iron all the advantage lies with the former.

The thickness wire used for the uncovered string is generally No. 24. In beginning the scaling of the bass strings, however, No. 17 or No. 18 are chosen to be notes nearest to the treble. The covering is usually from No. 25 to No.28 (standard note music, wire gadge), according to the size of the piano and the practicable string length. Of course, longer string maybe covered with higher wore.

The "first" covered string is generally approximately one-sixth shorter than the string immediately above it. This proportion, as suggested above may, however, is profitably disregarded if it thereby were possible to lengthen the bass string. These are always treated. These strings to each tone and he thickness of covering wire must be progressively increased as the scale descends. A descending increase «H one number in thickness of the covering wire for each pair of strings may properly he allowed unless the lengths are too closely alike or vice-versa, in which case suitable modifications may be made. Assuming that the descending lengths are arranged in arithmetical progression with a man of % of an inch, and supposing the highest covered string to be 45 inches long; then the suggested thickness should under all circumstances hold good. It may often be found, however, that the space of limitations of an instrument or other practical considerations make it impossible to follow out these rules with exactitude. In any case, we must remember that all such rules are themselves she fruit of empirical observation we must look when it becomes necessary to revise them ignored to satisfy the requirement of some particular situation.

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