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The Effect of Spherical and Non-Spherical Disturbances on OLSA and OLS Estimation Method: An Optimal Choice Using Monte Carlo

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Abstract

This paper undertakes a comparative analysis of estimation efficiency of Ordinary Least Square and Ordinary Least Square-Adjusted (OLSA) in a small sample when the economic model contains a spherical and non- spherical disturbance. A covariance matrix estimator that can consistently estimate the covariance of the model parameters which have been receiving attention in the econometric literature in recent time is employed But so much attention has been paid only to the asymptotic property, while pure OLS has continued to be the dominating estimation technique in small samples even in the face of non-spherical errors. This paper examines the small sample properties of OLS-Adjusted for moving average (MAJ, auto regression (AR), auto regressive moving average(ARMAJ .when the sample size is small and there' is a spherical and non-spherical error term. . Since Monte Carlo experiment remains one of the best approaches for empirical estimation of, finite sample properties It is herein employed in examining the small sample properties of OLSA compared to OLS. It is found that when the sample size was deliberately made small and the errors fixed at 0.4, 0,6, and 0.8 at lower error, OLS dominates and as the error increases OLSA dominates.

Keyword; Spherical and Non-spherical, OLS, OLSA

Econometric data are scarcely free of spherical and non-spherical disturbances, especially! when time series data .are involved. In almost all. cases the covariance structure are unknown, consequently the form of autocorrelation, heteroscedasticity and non-spherical disturbance is unknown. In such cases, model parameters can typically still be :"estimated consistently using the usual estimating functions but for valid inference in such models, a consistent covariance matrix estimate is essential. Over the

past 10 years several procedures for non-spherical consistent covariance estimation have been suggested in the econometric literature for the adjustment of OLS Adiele [2017]

It is important to recall that under non- spherical disturbance the usual OLS estimators although Linear, unbiased, and asymptotically, normally distributed are no longer minimum variance among all linear unbiased estimators. However the effect of spherical and non-spherical disturbances is yet to be investigated.

In short they are not efficient relative to other linear and unbiased estimator, that is, they may not be best, linear, unbiased, efficient (BLUE) and as a result, the usual t, F and $\boldsymbol{i}$ may not be valid.

However, in large sample, it is appropriate to use the Newey-West method to obtain standard errors of OLS estimators that are corrected for non-spherical error Adiele [2017]. This method is actually an extension of White's consistent standard errors method for large sample and may not be appropriate in small samples, since researchers have not shown interest in investigating the small sample properties. This paper wish to investigate this. Thus, it is known that when the sample is reasonably large, the Newey-West procedure to correct OLS for standard errors is employed not only in situations of-autocorrelation but also in cases of heteroskedasticity, and nonspherical for the extended HAC method can handle all Chalk [2014]

In the presence of autocorrelation, OLS estimators, although unbiased consistent and asymptotically normally distributed, are not efficient.

Therefore, the usual inference procedure based on the t, f and χ: test is no longer appropriate. On the other hand OLS-Adjusted procedure estimators are efficient, but the finite or small - sample, properties of these OLS-Adjusted estimators might actually do worse than OLS especially in the presence of spherical and non-spherical disturbances.

This paper seek to provide a well documented small - sample properties of OLSA compared to OLS especially at various degrees/levels of (p) spherical and nonspherical disturbances by employing the Monte-Carlo method - since Monte Carlo are particularly useful to study the behavior of estimators in small or finite samples. In a similar work Griliches and Rao [1969), carried out a Monte Carlo study to compare OLS and FGLS and found that if the sample is relatively small and the coefficient of the autocorrelation, p, is less than 0.3, OLS is good or better than FGLS, in the spirit of their study we investigate OLS and OLSA since OLSA is dominating in today's econometric literature for large samples.

The linear regression model

To fit notations, we considered the linear regression model

$Y_t = X^T I\ \beta + \mu_i (i = 1, \dots\dots, n)$ (1)

With dependent variable Yi k- dimensional regressor x_i with coefficient vector β and error term μ_i. In the usual matrix notation comprising all n observations this can be formulated as $y = X\beta + u$.

In the general linear model, it is typically assumed that the errors have zero mean and variance Var [u] = Ω.

Suitable regularity conditions. Adiele [2012], Green [1993], White [2000] the coefficients β can be consistently estimated by OLS giving the well-known OL'S Estimator $\hat{\beta}$ with corresponding OLS residuals $\hat{U}_{i:}$

$$\beta = (X^T X)r^{-1} X^T y, \qquad (2)$$

$$\hat{U} = (I_n - H)\, U = I_n - X(X^T X)^{-1} X^T)U, \qquad (3)$$

Where I_n is the n-dimensional identity matrix and H is usually called hat matrix. The estimate $\hat{\beta}$ are unbiased and asymptotically normal [4]. Their covariance matrix Ψ is usually denoted in one of the two following ways:

$$\Psi = \text{Var}\ [\beta] = X(X^T X)^{-1} X^T\ \Omega X\ (X^T)^{-1},$$

$$= \frac{1}{n} (X^T X)^{-1} \frac{1}{n}\ \theta\ \frac{1}{n} (X^T X)^{-1}, \qquad (5)$$

Where $\theta = \frac{1}{n} X^T\ \Omega\ X$ is essentially the covariance matrix of the scores or estimating n

functions $V_t(\beta) = X_i\ (y_i - x_i^T\ \beta)$. The estimating functions evaluated at the parameter

estimates $V_i = V_i(\ \hat{\beta}\ )$ have then sum zero.

For inference in the linear regression model, it is essential to have a consistent estimator Ψ. What kind of estimator should be used for Ψ depends on the assumptions about Ω. In the classical linear model independent and homoscedastic errors with o^2 arc assumed yieldin$_y$ n = o² In and

'I' = (i (X .X)' which can be consistently estimated by plugging ı11 the usual OLS estimators cr^2 = (n - k)"1 L:ni=1 G I ~. But if the independence and/ or nonspherical assumption is violated inference based on this estimators 'Vc(Jnst == 0² (X^f xr' will be biased. OLSA estimators tackle this problem by plugging an estimate 0. or (I> into (4) or (5) which are consistent in the presence of heteroskedasticity, autocorrelation, and nonspherical respectively. Such estimators and their implementation »re described in the following section.

Dealing with the joint effect of spherical and non-spherical disturbances

If the error terms *UI,* are not independent, n is not diagonal and without further specification of a parametric model for the type of independence it is typically burdensome to estimate n directly.

However, if the form of disturbances, spherical and non-spherical, hcteroskedasticity and autocorrelation is known, a solution to this problem is to estimate IT) instead which is essentially the covariance matrix of the estimating functions. This is what OLSA estimators do: Ψ_{OLSA} is computed by plugging an estimate Φ into equation (5) with

$$\Phi = \frac{1}{n}\sum_{j=1}^{n} W_{i-j} V_i V_j^T \qquad (6)$$

Where $w = (w_0 \ldots w_{n-1})^T$ is a vector of weights. An additional finite sample adjustment can be applied by multiplication with $n/(n-k)$, for many data structures. However, if their covariance matrix Ω is diagonal but has nonconstant diagonal elements OLSA handles this by plugging an estimate of type $\Omega = \text{diag}(w_1,\ldots,w_n)$ into equation (4) . this estimate will differ in their choice of the w_i,

Where $w_i = \frac{n}{n-k}\, \hat{u}_i^2$.

The error term is nonspherical because under the classical assumptions a joint confidence region for Σ would be an n-dimensional hypersphere. Then, it is a reasonable assumption that the nonspherical term should decrease with increasing $\rho = [i - j]$, otherwise β can typically not be estimated consistently by OLS – so that it is rather intuitive that the weight $W\rho$ should also decrease. Different choices for the vector of weights w have been placed by White [2000] in a more general framework of choosing the weights by kernel functions. Andrews [1991] showed that the bias of the estimators can be reduced by pre weighting the estimating functions V_i using a vector auto regression (VAR) of order ρ and applying the estimator in equation (6) to the Var (ρ) residuals subsequently. Andrews [1992] suggests an adaptive weighting scheme where the weights are chosen based on the estimated autocorrelation of the residuals u.

All the estimators mentioned above are of the form (6), i.e., a weighted sum of lagged products of the estimating functions corresponding to a fitted regression model. Therefore, it is reasonable to apply OLSA estimators, even when $w_i = \frac{n}{n-k}\, \hat{u}_i^2$. with the error term being spherical

Methodology

The basic model employed in our experiment is the simple regression model where the error term are jointly spherical and non-spherical. This model is given by

$$Y_t = \beta_0 + \beta_1 X_t + u_t \qquad (8)$$

where $u_t = \rho U_{t-1} + \epsilon_t$;

ϵ_t = white noise, $X_t = \lambda X_{t-1}$,

And $\lambda = 0.8$. This independent variable is commonly used in applied work and has already been used in other Monte Carlos studies to facilitate comparison of our results with pervious findings.

The values of the parameters β_0, β_1 in our model $Y_t = \beta_0 + \beta_1 X_t + u_t$ are arbitrarily chosen to be (1, 1). To obtain the X variable with real life quality, the following procedures are followed $X_t = 0.8\ X_{t-1}$ and the initial value X_0 is fixed arbitrarily as 515. This was popular model used by Adiele [2010], Lumley and Heagerty [1999]

In order to generate multivariate normal vectors to be use, the E-View software was used. Successive values of e_t drawn from N (0, 1) were used to calculate $U_t = \rho U_{t-1} + \epsilon_t$ and the first sample was thrown away to avoid problems of initial value. 10 sample of size 1000 each were generated for ρ, estimates of rho were used at 0.4, 0.6, 0.8.

After generating the error term for each specification, multivariate normal Y vectors are generated using the equation (8) The computations are made using E-View. The Y, X and U_t is used to re-estimate the β's. In our study, we employ the minimum Bias minimum variance and mean square error criteria to evaluate and compare the estimators.

Result

Table 1 result of simulation when € = 0.4

n = 20,	€ = 0.4,	Average Bias		
Estimators	**β_0**	**β_1**	**β_2**	**SBIAS**
OLS	-0.22386030	-0.16374834	0.0119032172153	-0.307705125
OLS-AR	-0.342653	-0.079086	0.032342	-0.40739609
OLS-MA	-0.5213881	-0.0880390	0.04985744	-0.55956966
OLS-ARMA	-0.1406302	-0.0948305	0.02080134	-0.21465936
n = 20,	**€ = 0.4,**	**Root Mean Square Error**		
Estimators	**β_0**	**β_1**	**β_2**	**SRMSE**
OLS	0.54180395	0.334649889	0.0784003547	1.105854194
OLS-AR	0.801326638	0.33193574	0.1064390123	1.32970139
OLS-MA	0.90596217073	0.361721572	0.105295703	1.383979446
OLS-ARMA	2.6723246236	0.363586640	0.136041548	3.171952811
n = 20,	**€ = 0.4,**	**Bias Minimum Variance**		
Estimators	**β_0**	**β_1**	**β_2**	**SVAR**
OLS	0.40041157693	0.120150527476	0.00782593041163	0.528388034
OLS-AR	0.64803995839	0.102244980914	0.0701498963917	0.760434835
OLS-MA	0.568974033817	0.123091626463	0.00860742069997	0.70066708
OLS-ARMA	7.12154203997	0.123202418701	0.0180746067583	7.262819064

Table 2 result of simulation when € = 0.6

n = 20,	€ = 0.6,	Average Bias		
Estimators	**β_0**	**β_1**	**β_2**	**SBIAS**
OLS	0.0128742463424	-0.276797476892	-0.0274362550795	0.291359485
OLS-AR	-0.492558106109	-0.0631466138879	0.0451418040974	0.510562915
OLS-MA	-0.527967050628	-0.0907706843381	0.0462759432267	0.572461791
OLS-ARMA	-0.3562088992	-0.103811945262	0.0271427111541	0.432878126
n = 20,	**€ = 0.6,**	**Root Mean Square Error**		
Estimators	**β_0**	**β_1**	**β_2**	**SRMSE**

OLS	0.796954134617	0.485209780696	0.124921789036	1.407085704
OLS-AR	0.779525805116	0.305224446783	0.0944420356545	1.179192287
OLS-MA	0.864434575218	0.343507820824	0.104019082855	1.311961479
OLS-ARMA	1.04000290124	0.380009348081	0.130827923478	1.550840172
n = 20,	€ = 0.6,	Bias Minimum Variance		
Estimators	β_0	β_1	β_2	SVAR
OLS	0.634970146463	0.158811688069	0.014852705283	0.808634539
OLS-AR	0.365046992949	0.0891744680684	0.00688151562139	0.461102976
OLS-MA	0.468497928283	0.109758305832	0.00867850667655	0.58693474
OLS-ARMA	0.954721254721	0.13363018465	0.0163792187928	1.104730658

Table 3 result of simulation when € = 0.8

n = 20,	€ = 0.8,	Average Bias		
Estimators	β_0	β_1	β_2	SBIAS
OLS	2.50486905171	-1.06441413878	-0.337022241376	1.103432672
OLS-AR	0.226617704516	-0.133082233081	-0.0363024786565	0.057232992
OLS-MA	0.464882941196	-0.3886413327773	-0.0773756847264	1.13407632
OLS-ARMA	2.06890286693	-0.471150674896	-0.235904781974	1.361847409
n = 20,	€ = 0.8,	Root Mean Square Error		
Estimators	β_0	β_1	β_2	SRMSE
OLS	3.3105783117	1.28814171469	0.425812888777	5.024532914
OLS-AR	3.09625091089	0.34888023005	0.31020203699	3.755333177
OLS-MA	1.36288023193	0.57984574679	0.172749633701	2.115469611
OLS-ARMA	5.23892017253	0.827133615208	0.542504130062	6.614137664
n = 20,	€ = 0.8,	Bias Minimum Variance		
Estimators	β_0	β_1	β_2	SVAR
OLS	4.68555979166	0.526331618274	0.0677326250664	5.279624034
OLS-AR	9.5354141192	0.104006534158	0.0949074337966	9.734328087
OLS-MA	1.64132637756	0.18564501413	0.0238533663973	1.850824757
OLS-ARMA	23.1659255013	0.462167058953	0.238659664976	23.86675222

Bias for estimators of β in the model

In our estimates, we observe that the SBIAS of OLS corrected for ARMA is lower than the SBIAS for OLS, OLS-AR, OLS-MA and also Lower than the SBIAS of the iv methods for n = 20 and ℓ = 0.4 It is observed that all the estimators are negatively biased for β_0 (1), when n = 20, and ℓ = 0.4. The pattern is mixed up for increasing values of ℓ. When n = 20 and ℓ = 0.6, it is observed that only OLS is positively biased while all other estimators are negatively biased for β_0 (.), When n = 20 and ℓ = 0.8, it is observed that all the OLS-estimators are negatively biased for β_0 (.) Here we also observed that the SBIAS for OLS-AR is lower than the SBIAS for all other estimators.

However, this result confirms the sequel, the word dominate is used to qualify a magnitude which is smaller than another one and therefore it is preferred. The meaning is going to remain the same throughout this work. From table … for n = 20, ℓ = 0.4 we notice that on the basis of minimum RMSE, SRMSE of OLS has slight edge over the SRMSE of OLS-AR, OLS-MA, OLS-ARMA, . But as ℓ increases from 0.4 to 0.6 for the same sample size, RMSE β_1(OLS) is 1.59 times higher than the RMSE of β_1 (OLS-AR), 1.41 times higher of β_1 (OLS-MA) 1.28 times higher than (OLS-ARMA) The RMSE of β_2 (OLS) for the same sample size and ℓ = 0.6 is 1.32 times higher than the RMSE of β_2 (OLS-AR), 1.20 times of the RMSE of β_2 (OLS-MA), However, the RMSE for β_2 (OLS-AR) is lower than all the RMSE of β_2 (0) of all other estimators. For the SRMSE properties, we notice that OLS-AR dominates all other estimators. By way of illustration we observe that the SRMSE of OLS 1.16, 1.41, and 5.02 for n = 20, ℓ = 0.4, 0.6, 0.8 respectively while the SRMSE of OLS-AR are 1.33, 1.17 and 3.76 From these estimates of SRMSE, we observed that OLS dominates only when ℓ = 0.4 while for increased ℓ, 0.6, 0.8 OLS-AR dominate all the estimators, which suggests that in this respect, the effect of increasing nonspherical error on the comparative performance of the estimators is significant.

It can therefore, be reasonably assumed that for small nonsperical error OLS dominate the OLS-AR, OLS-MA, OLS-ARMA, while for large autocorrelation OLS-AR dominate OLS, OLS-MA, OLS-ARMA, In our summary, the major conclusions which could be drawn from our experiment when the OLS and OLSA models are involved are the following.When ℓ = 0.4 OLS-ARMA dominate, when ℓ = 0.6 OLS dominate and when ℓ = 0.8 OLS-AR dominate all others respectively.

On the basis of VAR. OLS dominate when ℓ = 0.4, OLS-AR dominate when ℓ = 0.6 and OLS-MA dominate when ℓ = 0.8 while on the basis of RMSE properties OLS dominate when ℓ = 0.4 OLS dominate whereas for increased nonspherical error OLS-AR dominates, suggesting that for small nonsperical error OLS dominates on the basis of VAR, and RMSE.In our estimates from Table 1, we observed that when $\rho = 0.4$, the SBIAS of OLS in lower than the SBAIS for OLSA and it is also observed that all the estimators are negatively biased for β_0, β_1.When Rho is increased to 0.6 ($\rho = 0.6$), it is observed that OLS is positively biased *f* or β_0, and negatively biased for β_1, whereas OLSA is negatively biased for β_0, β_1.When $\rho = 0.8$, it is observed that both estimators are positively biased for β_0 and both negatively biased for β_1. Here we observed that when ρ is small, the SBAIS for OLS is lower that SBIAS for OLSA suggesting that OLS dominates. But for increase (ρ = 0.6, 0.8) the SBIAS for OLSA is lower suggesting that OLSA dominates. The word dominate is used to quantity a magnitude which is smaller than another one and therefore it is preferred, the meaning is going to remain the same throughout.

Conclusion

This study examined a specification of independent variable often associated with Linear models when the error terms take any shape spherical or non-spherical. Attention is focused on the ordinary least square (OLS) method and on Newy West spherical and non-spherical heteroskedasticity and autocorrelation consistent estimation method for OLSA. The specification of the independent variable considered is such that the data are generated so that they resemble data obtained from control1ed laboratory experiment. The primary emphasis of the study has been on examining the sampling properties of OLSA and OLS when the sample size is relatively smal1and the error term jointly spherical and non-spherical. Monte Carlo experiment yield results which are of practical and methodological relevance in conducting empirical studies, the Monte Carlo evidence shows that the effect of increasing non-spherical or spherical error terms on the comparative performance of the estimators are remarkably significant. It is difficult to make a conclusive statement regarding the performance which depends on a number of factors including the size of error and on the individual coefficient being estimated.

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